Ayudantía 3 MAT 1107

① a)

	Ase.	Ens.
M:	3	1
S:	2	2

x_1: Tiempo Aserrando mesas
x_2: Tiempo Aserrando sillas
y_1: Tiempo Ensamblaje mesas
y_2: Tiempo Ensamblaje sillas

Máximo: $3x_1 + 2x_1 \leq 12$
$y_1 + 2y_2 \leq 8$

O Sea: Ensamblan máx entre 8 mesas y 0 sillas hasta 0 mesas y 4 sillas
Aserran máx entre 4 mesas y 0 sillas hasta 0 mesas y 6 sillas

Casos límites no entran

∴ No pueden producir más de 4 mesas completas al día ni tampoco más de 4 sillas. Quedan las siguientes combin.

(mesas, sillas): (0,0) (0,1) (1,1) (2,1) (2,2)
(1,0) (0,2) (1,2) (3,1)
(2,0) (0,3) (1,3)
(3,0) ~~(0,4)~~
~~(4,0)~~

b) $\sqrt{x^2+1} > 2 \Rightarrow x^2+1 > 4 \Rightarrow x^2 > 3 \ / \sqrt{\ }$

$\Rightarrow |x| > 3 \quad \therefore \quad x > \sqrt{3} \ \vee \ x < -\sqrt{3}$

c) $|x+1| \leq \sqrt{x^2-4x+4} + 3$

Ambos lados positivo: $(x+1)^2 \leq x^2 - 4x + 4 + 6 \cdot \sqrt{x^2-4x+4} + 9$

$\Rightarrow$ ~~x^2~~ $+ 2x + 1 \leq$ ~~x^2~~ $- 4x + 13 + 6 \cdot (x-2) \quad / + 4x - 13$

$6x - 12 \leq 6x - 12$

$x \leq x$ Se cumple para todos los reales ($\mathbb{R}$)

$\therefore x \in \mathbb{R}$

② a) $|5x+5|-8 \leq 17$

$|5x+5| \leq 25 \Rightarrow -25 \leq 5x+5 \leq 25$

$-30 \leq 5x \leq 20 \quad \therefore \; -6 \leq x \leq 4 \Rightarrow x \in [-6;4]$

b) $\left|\frac{x^2+5x+4}{x^2-4x-5}\right| > 2 \Rightarrow \left|\frac{(x+4)\cdot(x+1)}{(x-5)\cdot(x+1)}\right| > 2 \Rightarrow \left|\frac{x+4}{x-5}\right| > 2$

C1: $\frac{x+4}{x-5} > 2 \Rightarrow x+4 > 2x-10 \Rightarrow 14 > x \quad S_1 =]-\infty, 14[$

C2: $\frac{x+4}{x-5} < -2 \quad | \cdot (x-5)$

a) $(x-5) < 0 \Rightarrow x+4 > -2x+10 \Rightarrow 3x > 6 \Rightarrow x > 2 \quad S_2 =]2;\infty[$

b) $(x-5) > 0 \Rightarrow x+4 < -2x+10 \Rightarrow 3x < 6 \Rightarrow x < 2 \Rightarrow (x-5) < 0$ ⚠

$\therefore \; x \in S_1 \cap S_2 =]2, 14[$

c) $|3x+2| \geq |x+1| + |2x+1| \Rightarrow |3x+2| - |x+1| - |2x+1| \geq 0$

$f(x)$	$-\infty$ C1 -1	C2 $-\frac{2}{3}$	C3 $-\frac{1}{2}$	C4 $+\infty$
$3x+2$	−	−	+	+
$x+1$	−	+	+	+
$2x+1$	−	−	−	+

C1: $x \in]-\infty, -1]$

$\Rightarrow -(3x+2) + (x+1) + (2x+1) \geq 0$

$\Rightarrow 0 \geq 0 \quad \therefore \; S_1 =]-\infty, -1]$

C2: $x \in]-1, -\frac{2}{3}] \Rightarrow -(3x+2) - (x+1) + (2x+1) \geq 0$

$\Rightarrow -2x-2 \geq 0 \Rightarrow x \leq -1 \quad \therefore \; S_2 =]-\infty, -1] \cap]-1, -\frac{2}{3}] = \emptyset$

C3: $x \in]-\frac{2}{3}; -\frac{1}{2}] \Rightarrow (3x+2) - (x+1) + (2x+1) \geq 0$

$\Rightarrow 4x+2 \geq 0 \Rightarrow x \geq -\frac{1}{2} \quad \therefore \; S_3 =]-\frac{2}{3}; -\frac{1}{2}] \cup [-\frac{1}{2}, \infty[= [-\frac{1}{2}]$

C4: $x \in]-\frac{1}{2}; \infty[\Rightarrow (3x+2) - (x+1) - (2x+1) \geq 0$

$\Rightarrow 0 \geq 0 \quad \therefore \; S_4 =]-\frac{1}{2}; \infty[$

$S_T = S_1 \cup S_2 \cup S_3 \cup S_4 =]-\infty; -1] \cup [-\frac{1}{2}; \infty[$

d) $|x^2-2x| + x\cdot|x-3| \geq 3 \Rightarrow |x|\cdot|x-2| + x\cdot|x-3| \geq 3$

	$-\infty$		0		2		3	$+\infty$
x		−		+		+		+
$x-2$		−		−				+
$x-3$		−		−		−		+

C1: $x \in (-\infty, 0)$

$\Rightarrow -x\cdot(-(x-2)) + x\cdot(-(x-3)) \geq 3$

$\Rightarrow x^2 - 2x - x^2 + 3x \geq 3$

$\Rightarrow x \geq 3 \quad \therefore S_1 =]-\infty, 0[\cap [3, \infty[= \emptyset$

C2: $x \in [0, 2[$

$\Rightarrow x\cdot(-(x-2)) + x(-(x-3)) \geq 3 \Rightarrow -x^2+2x-x^2+3x \geq 3$

$\Rightarrow -2x^2+5x-3 \geq 0 \Rightarrow 2x^2-5x+3 \leq 0 \Rightarrow (2x-3)\cdot(x-1) \leq 0$

a) $2x-3 \leq 0 \wedge x-1 \geq 0$

$\Rightarrow x \leq 3/2 \wedge x \geq 1$

$\Rightarrow x \in [1, 3/2]$

b) $2x-3 \geq 0 \wedge x-1 \leq 0$

$x \geq 3/2 \wedge x \leq 1$

$x \in]-\infty; 1] \cap [3/2, \infty[= \emptyset$

$S_2 = [1; 3/2] \cap [0; 2[\Rightarrow [1; 3/2]$

C3: $x \in [2, 3[$

$\Rightarrow x\cdot(x-2) + x\cdot(-(x-3)) \geq 3 \Rightarrow x^2-2x-x^2+3x \geq 3$

$\Rightarrow x \geq 3 \quad : S_3 = [2, 3[\cap [3, \infty[= \emptyset$

C4: $x \in [3, \infty[$

$\Rightarrow x\cdot(x-2) + x\cdot(x-3) \geq 3 \Rightarrow x^2-2x+x^2-3x \geq 3$

$\Rightarrow 2x^2-5x-3 \geq 0 \Rightarrow (2x+1)(x-3) \geq 0$

$\Rightarrow$ a) $2x+1 \geq 0 \wedge x-3 \geq 0$

$x \geq -1/2 \wedge x \geq 3$

$x \in [3, \infty[$

b) $2x+1 \leq 0 \wedge x-3 \leq 0$

$x \leq -1/2 \wedge x \leq 3$

$x \in]-\infty, -1/2] \cap [3, \infty[= \emptyset$

$S_4 = [3, \infty[$

$\therefore S_T = S_1 \cup S_2 \cup S_3 \cup S_4 = [1; 3/2] \cup [3; \infty[$

③ $z>0$: $z \in \mathbb{R}$ A_z = cjto solución

$0 < z_1 < z_2$ Demostrar que $A_{z_1} \subset A_{z_2}$

$$|x^2+zx+z^2| \leq zx+2z^2$$

C1: $x^2+zx+z^2 \leq zx+2z^2$

$$x^2 - z^2 \leq 0 \Rightarrow (x+z)\cdot(x-z) \leq 0$$

$\Rightarrow x+z \leq 0 \wedge x-z \geq 0 \quad \vee \quad x+z \geq 0 \wedge x-z \leq 0$

$x \leq -z \qquad x \geq z \qquad x \geq -z \wedge x \leq z$

$x = z \qquad x \in [-z, z]$

C2: $x^2+zx+z^2 \geq -(zx+2z^2)$

$$x^2+zx+z^2 \geq -zx-2z^2 \quad /+zx$$

$$\Rightarrow (x+z)^2 \geq -2z^2$$

Luego, $(x+z)^2 \geq 0 \quad \forall x \in \mathbb{R} : z \in \mathbb{R}^+$

Además, $-z^2 \in \mathbb{R}^-$

$\therefore (x+z)^2 \geq -2z^2 \quad \forall x \in \mathbb{R}$

Luego, el cjto sol. es $\mathbb{R} \cap [-z, z] \Rightarrow x \in [-z, z]$

Si evaluamos en z_1 y z_2, tenemos

$A_{z_1} = [z_1, z_1]$; $A_{z_2} = [-z_2, z_2]$ Como $z_1 < z_2$ entonces:

$-z_1, z_1 \in A_{z_2} \therefore A_{z_1} \subset A_{z_2}$

④

a) $\sqrt{x^2-4} \leq x$

Primero, tenemos que hacer que $x^2-4 \geq 0 \quad \therefore x^2 \geq 4 \Rightarrow x \geq 2 \vee x \leq -2$ (1)

Luego, $\sqrt{x^2-4} \leq x \;/\,(\,)^2 \Rightarrow |x^2-4| \leq x^2$

$\Rightarrow -x^2 \leq x^2-4 \leq x^2 \Rightarrow -2x^2 \leq -4 \leq 0 \quad : -4 \leq 0 \;\forall x \in \mathbb{R}$ (2)

$\Rightarrow -x^2 \leq -2 \Rightarrow x^2 \geq 2 \quad \therefore x \geq \sqrt{2}$ (3)

$\therefore x_1 \in [2, \infty[\;(1) \cap \mathbb{R} \;(2) \cap [\sqrt{2}, \infty[\;(3) = [2, \infty[$

$\vee \quad x_2 \in]-\infty, 2] \cap \mathbb{R} \cap [\sqrt{2}, \infty[= [\sqrt{2}, 2] \in [2, \infty[$

$\therefore x \in [2, \infty[$ //

b) $\sqrt[3]{x^3-8} \leq x \;/\,(\,)^3$

$x^3-8 \leq x^3 \Rightarrow -8 \leq 0 \quad \therefore x \in \mathbb{R}$ //

Parece que la paridad del exponente afecta al conjunto solución.

⑤ a) $a^2+d^2=1=c^2+d^2$

$ac+bd \leq 1$

$(a-c)^2 \geq 0 \Rightarrow a^2+c^2-2ac \geq 0 \Rightarrow a^2+c^2 \geq 2ac$

$(b-d)^2 \geq 0 \Rightarrow b^2+d^2 \geq 2\cdot bd$

$\therefore (a-c)^2+(b-d)^2 \geq 0 \iff a^2+b^2+c^2+d^2 \geq 2(ac+bd)$

$\therefore a^2+b^2+c^2+d^2=2 \Rightarrow 2/2 \geq ac+bd$

$\Rightarrow ac+bd \leq 1$ //

b) $x \in \mathbb{R}^+$

$x^3+\frac{1}{x^3} \geq x+\frac{1}{x} \Rightarrow x^3+\frac{1}{x^3}-x-\frac{1}{x} \geq 0$

$\Rightarrow (x^3-x)+\left(\frac{1}{x^3}-\frac{1}{x}\right) = x(x^2-1)-\left(\frac{x^2-1}{x^3}\right)$

$\Rightarrow (x^2-1)\cdot(x-\sqrt{x^3}) = \frac{(x^2-1)}{x^3}\cdot(x^4-1) = \frac{(x^2-1)\cdot(x^2-1)\cdot(x^2+1)}{x^3}$

$= \frac{(x^2-1)^2\cdot(x^2+1)}{x^3}$

Luego: $(x^2-1)^2 \geq 0 \quad \forall x \; ; \; (x^2+1)>0 \; : \; x^2>0$

y $x^3>0$ porque $x>0$

$\therefore \frac{(x^2-1)^2\cdot(x^2+1)}{x^3} \geq 0$

$\Rightarrow x^3+\frac{1}{x^3} \geq x+\frac{1}{x}$ //

⑥ $a^2 \geq 1$, $b^2 \geq 4$ $\quad \frac{ab-b}{2} + a \leq 1 \Rightarrow ab - b + 2a - 2 \leq 0$ ✱

Demostrar $ab - 2a + b \leq 2$

$a^2 - 1 \geq 0 \Rightarrow (a+1)\cdot(a-1) \geq 0$

$b^2 - 4 \geq 0 \quad\quad (b+2)\cdot(b-2) \geq 0$

$\Rightarrow (a+1)\cdot(a-1)\cdot(b+2)\cdot(b-2) \geq 0$

$\Rightarrow (a+1)\cdot\underbrace{(ab+2a-b-2)}_{\text{✱}\ \geq 0}(b-2) \geq 0$

$\Rightarrow (a+1)\cdot(b-2) \geq 0$

$ab - 2a + b - 2 \geq 0 \Rightarrow ab - 2a + b \geq 2$