RESUMEN MAT 1203 - INTRO. ÁLGEBRA

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LICC - 623

ESTEBAN ORTEGA

Indice

- Tema 1 : LÓGICA
- Tema 2 : POLINOMIOS
- Tema 3 : TRIGONOMETRÍA
- Tema 4 : GEOMETRÍA ANALITICA

· LÓGICA

p	q	p ∨ q
V	V	V
V	F	V
F	V	V
F	F	F

p	q	p ∧ q
V	V	V
V	F	V
F	V	V
F	F	F

p	q	p → q
V	V	V
V	F	F
F	V	V
F	F	V

p	q	p ↔ q
V	V	V
V	F	F
F	V	F
F	F	V

· Propiedades (Proposiciones)

① Morgan : $\neg(p \wedge q) = \neg p \vee \neg q$

② Conmutativa : $p \vee q = q \vee p$

③ Asociativa : $p \vee (q \vee r) = (p \vee q) \vee r$

④ Distriuutiva : $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

⑤ Equivalencia : $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

⑥ Contrapositiva : $p \rightarrow q = \neg q \rightarrow \neg p$

Demostraciones:

① Directa: $P \to q$

① Asumir $P = V$
② Concluir $q = V$

ej: a par $\to$ a^2 par

$a = 2k, k \in \mathbb{Z}$ ↗

$a^2 = 4k^2$
$a^2 = 2(2k^2)$
$a = 2M$ $\therefore V$

② Contradicción: $P \wedge \neg q \to F$

① Negamos la tesis
② Usamos la Hipotesis
③ Llegamos a algo Falso

ej: a^2 par $\to$ a par

$\neg q$: a NO es par

$a = 2k+1$ $/(\,)^2$ $= a^2 = 4k^2 + 4k + 1$
$a^2 = 2(2k^2 + 2k) + 1$

Hip: $a^2 = 2M$ ⇏ $a^2 = 2M + 1$ $\therefore V$

③ Contrapositiva: $p \to q \equiv \neg q \to \neg p$

① Negar tesis
② Llegar a la No Hipotesis

ej: a par $\to$ a^2 par

$\Leftrightarrow$ a^2 impar $\to$ a impar

④ Inducción: Demostrar propiedad $\forall n \in \mathbb{N} \to P(n)$

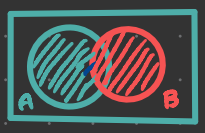
① Caso Base: Demostrar $P(1) = V$
② Caso INDUCTIVO: ① Asumir $P(n) = V$
② Demostrar $P(n+1)$ //

CONJUNTOS

$A = \{1, 2, 3, \dots\}$

- **Igualdad**: $A = B \Leftrightarrow$ mismos elementos

ej
$A = \{1, 2, 3\}$
$B = \{3, 2, 1\}$

- **Vacio**: $\emptyset = \{\}$ No elem
- **Universo**: $U \rightarrow$ 

$U \rightarrow$ todos los elem
- **Subconjuntos**: $A \subseteq B \Leftrightarrow x \in A \rightarrow x \in B$



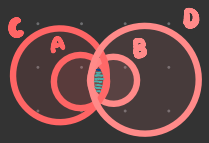
$A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$

- **Cardinalidad**: nº de elem : $n(A)$ o $\#A$

ej $\#\{1,1,2,3\} = 3$
$\#\{\emptyset, \{\emptyset\}\} = 2$
$\#\{\emptyset\} = 1$

- **Logica**: $\cdot\ A \cap B = \{x \in A \wedge x \in B\}$
$\cdot\ A \cup B = \{x \in A \vee x \in B\}$

- **Corolario**
  1) $A \cap B \subseteq A \subseteq A \cup B$
  2) $(A \subseteq B) \wedge (A \subseteq C) \rightarrow A \subseteq (B \cap C)$
  3) $(A \subseteq B) \wedge (C \subseteq D) \rightarrow (A \cup C) \subseteq B$
  4) $(A \subseteq C) \wedge (B \subseteq D) \rightarrow A \cap B \subseteq C \cap D$
  $\rightarrow A \cup B \subseteq C \cup D$



- **Complemento:** $A^c = x \in U : x \notin A$

- **Diferencia:** $A - B = x \in A \wedge x \notin B$



$$A - B = A \cap B^c$$

- **Dif. Simetrica:** $A \triangle B = (A - B) \cup (B - A)$

↳ $A \triangle B = (A \cup B) - (A \cap B)$



- **Cuantificadores**

1) $\forall x \in U$: Para todo x
2) $\exists x \in U$: Existe por lo menos 1
3) $\exists! x \in U$: Existe Solo 1

- **Negación:**

1) $\neg(\forall x \in U)(P(x)) = \exists x \in U\ (\neg P(x))$

2) $\neg(\exists x \in U)(P(x)) = \forall x \in U\ (\neg P(x))$

3) $\neg(\exists! x \in U)(P(x)) = (\forall x \in U)(\neg P(x)) \vee (\exists y \in U)(y \neq x \wedge P(y))$

POLINOMIOS

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots\dots + a_1 x + a_0$$

↳ $P(x) = \sum_{i=0}^{n} a_i x^i$

- **Grado**: $g(p(x)) = n$
- **Cuerpo**: $P \in \mathbb{R}[x] \rightarrow$ Coef $\in \mathbb{R}$
  $P \in \mathbb{C}[x] \rightarrow$ Coef $\in \mathbb{C}$
- **Raiz**: α Raiz $\Leftrightarrow$ $P(\alpha) = 0$
- **Factorización**: $P(x) = (x - a)\, Q(x)$
- **División Euclidiana**:

$(x^2 + 2x + 1) \div (x + 1) = x + 1 \rightarrow$ Cociente

$$\begin{array}{r|l} -x^2 + x & \\ \hline & x + 1 \\ -x + 1 & \\ \hline 0 \rightarrow \text{Resto} & \end{array}$$

- **Teorema Resto**

$P(x) = Q(x)(x - \alpha) + R(x)$

$P(\alpha) =$ Resto

· Ruffini :

$P(x) = x^2 + 2x + 1 \quad \alpha = -1\ ?$

	x^2	$2x$	1
	1	2	1
1	↓ $+1$	-1	-1
	1	1	0 → Resto 0 : $x - \alpha$ Divisor
	$(x+1)$: Cuociente		↓ $(x+1)$

· Teorema Ceros Racionales :

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots\dots + a_1 x + a_0$$

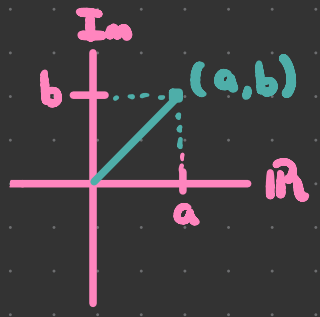
↓

Posible raiz :

$$\alpha = \frac{P}{q} = \frac{\{\text{Divisores } a_0\}}{\{\text{Divisores } a_n\}}$$

· Numeros Complejos $\mathbb{C}$

- $z = a + bi \quad a, b \in \mathbb{R}$
- $\overline{z} = a - bi$
- $|z| = \sqrt{a^2 + b^2}$



· Teo. Fundamental Algebra

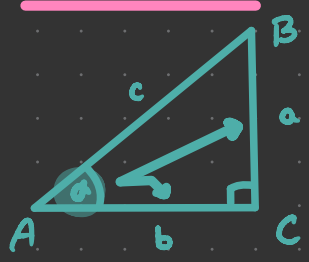
Si: $P \in \mathbb{C}_{[x]} \wedge g(P) > 1 \longrightarrow \exists\, \alpha \in \mathbb{C}$

· Teorema Raiz Conjugada :

Si: $P \in \mathbb{R}_{[x]} \wedge \alpha$ raiz $\in \mathbb{C} \longrightarrow \overline{\alpha}$ es Raiz

• TRIGONOMETRÍA

① Razones:



$\sin\alpha = \frac{a}{c} \rightarrow \operatorname{cosec}\alpha = \frac{c}{a}$

$\cos\alpha = \frac{b}{c} \rightarrow \sec\alpha = \frac{c}{b}$

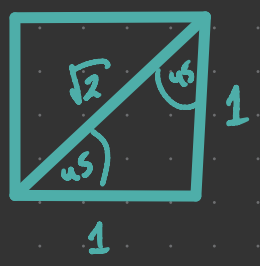
$\tan\alpha = \frac{a}{b}$ $\quad \cot\alpha = \frac{b}{a}$

② Identidades:

1) $\text{sen}^2 + \cos^2 = 1$
2) $tg^2 + 1 = \sec^2$
3) $1 + \cot^2 = \operatorname{cosec}^2$

③ Angulos Notables:

A)



B)



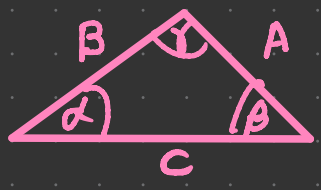
tabla

	0	30	45	60	90
sen	0	1	2	3	4
cos	4	3	2	1	0

Z

④ Teorema Seno:

$$\frac{\text{Sen}\,\alpha}{A} = \frac{\text{Sen}\,\beta}{B} = \frac{\text{Sen}\,\gamma}{C}$$



⑤ Teorema Coseno:

- $A^2 = B^2 + C^2 - 2BC\cos\alpha$
- $B^2 = A^2 + C^2 - 2AC\cos\beta$
- $C^2 = A^2 + B^2 - 2AB\cos\gamma$

6) Formulas

- **Sumas** :

$$\operatorname{sen}(a \pm \beta) = \operatorname{Sen}(\alpha)\cos(\beta) \pm \operatorname{sen}(\beta)\cos(\alpha)$$

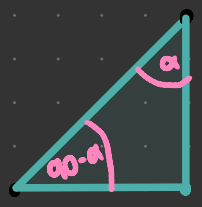
$$\cos(a \pm \beta) = \cos(\alpha)\cos(\beta) \mp \operatorname{sen}(\alpha)\operatorname{sen}(\beta)$$

$$\tan(a \pm \beta) = \frac{tg(\alpha) \pm tg(\beta)}{1 \mp tg(\alpha)\cdot tg(\beta)}$$

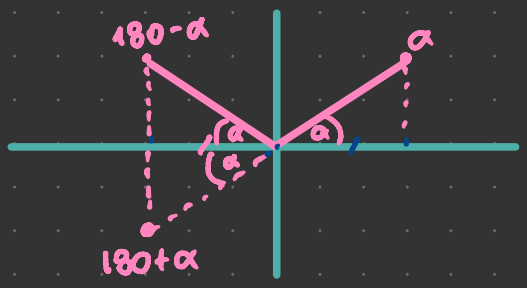
$a + a$

- **Angulo Doble** :
  - $\operatorname{Sen}(2\alpha) = 2\operatorname{sen}(\alpha)\cos(\alpha)$
  - $\cos(2\alpha) = \cos^2(\alpha) - \operatorname{sen}^2(\alpha)$
  - $tg(2\alpha) = \dfrac{2\,tg(\alpha)}{1 - tg^2(\alpha)}$

- **Redox** :
  - $\operatorname{sen}(90 - \alpha) = \cos\alpha$
  - $\cos(90 - \alpha) = \operatorname{sen}\alpha$
  - $tg(90 - \alpha) = ctg\,\alpha$
  - $ctg(90 - \alpha) = tg\,\alpha$



- **Redox** :
  - $\operatorname{sen}(180 - \alpha) = \operatorname{Sen}\,\alpha$
  - $\operatorname{Sen}(180 + \alpha) = -\operatorname{sen}\,\alpha$
  - $\cos(180 - \alpha) = -\cos\alpha$
  - $\cos(180 + \alpha) = -\cos\alpha$



- **Prostaferisis** :

$$\operatorname{Sen}x + \operatorname{sen}y = 2\operatorname{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\operatorname{Sen}x - \operatorname{sen}y = 2\cos\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{x-y}{2}\right)$$

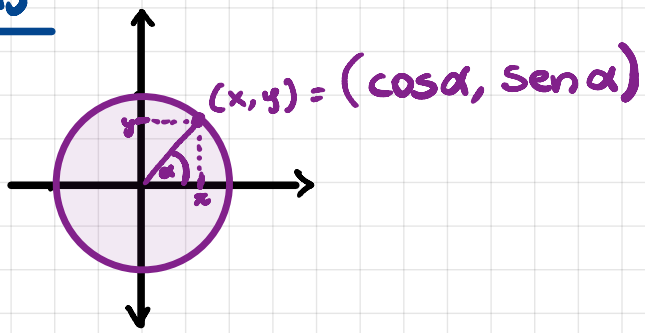
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\operatorname{sen}\left(\frac{x+y}{2}\right)\operatorname{Sen}\left(\frac{x-y}{2}\right)$$

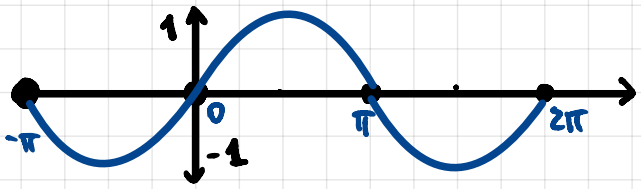
- $\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta) = 2\operatorname{sen}\alpha\cos\beta$
- $\operatorname{sen}(\alpha+\beta) - \operatorname{sen}(\alpha-\beta) = 2\cos\alpha\operatorname{sen}\beta$

Funciones Trigonometricas

1) Circunferencia Unitaria :



∘ Función Seno: $y = \text{sen}(x)$

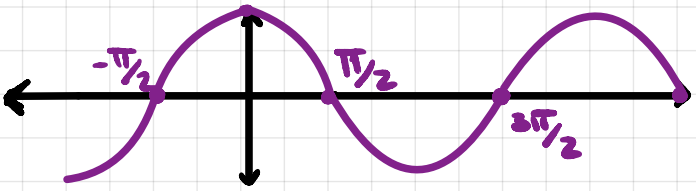


1) Periodo: $2\pi \rightarrow \text{Sen}(x \pm 2\pi) = \text{Sen}x$

2) Acotado: $[-1, 1]$

3) Impar: $\text{Sen}(-x) = -\text{sen}(x)$

∘ Función Coseno: $y = \cos(x)$

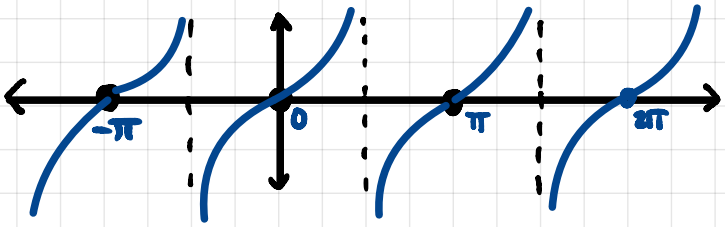


1) Periodo: $2\pi \rightarrow \cos(x \pm 2\pi) = \cos x$

2) Acotado: $[-1, 1]$

3) Par: $\cos(-x) = \cos x$

∘ Función tang: $y = tg(x)$



1) Periodo: $\pi \rightarrow tg(x \pm \pi) = tgx$

2) No Acotado

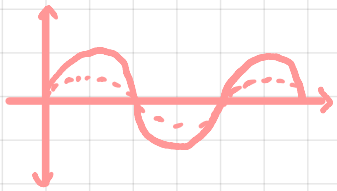
3) Impar: $tg(-x) = -tg(x)$

- **Transformaciones**

$$y = a \cdot \operatorname{sen}(K \cdot x - b) + h$$

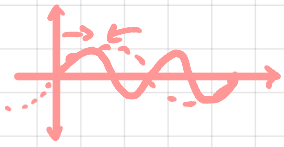
1) a = amplitud
2) $\frac{2\pi}{K}$: Periodo
3) b, h = Defase
4) $\frac{K}{2\pi}$ = Frecuencia

ej: $2\operatorname{sen} x$



ej: $\operatorname{sen}(2x)$

Periodo: $\frac{2\pi}{2} = \pi$



- **Forma Polar**:

$$z = a + bi \rightarrow r \cdot \operatorname{cis}(\theta)$$

- $r = \sqrt{a^2 + b^2} = |z|$
- $\operatorname{cis}\theta = \cos\theta + i\operatorname{sen}\theta$

1) $z_1 \cdot z_2 = r_1 \cdot r_2 \cdot \operatorname{cis}(\theta_1 + \theta_2)$
2) $z_1^n = r^n \operatorname{cis}(\theta \cdot n)$

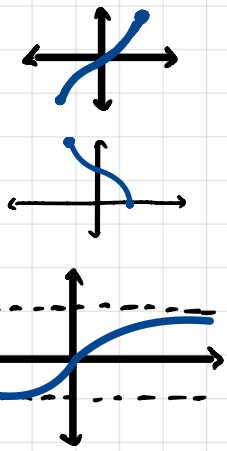
- **Teorema Moivre**: $\operatorname{cis}(\theta)^n = \cos(n\cdot\theta) + i\operatorname{sen}(n\cdot\theta)$

!! - **Raices de la Unidad**: $z^n = a$

$$\Rightarrow z^n = |r| \cdot \operatorname{cis}(\theta + 2\pi K)$$

$$z = \sqrt[n]{r}\,\operatorname{cis}\left(\frac{\theta + 2\pi K}{n}\right), \quad K = \{0, 1, 2, \ldots, n-1\}$$

• Inversas Trigonometricas



- $\text{Arcsen } y = x \iff \text{sen } x = y \;,\; x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\text{Arccos } y = x \iff \cos x = y \;,\; x \in [0, \pi]$
- $\text{Arctg } y = x \iff \text{tg } x = y \;,\; x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

• Compuestas Inversas

- $\text{Arcsen}(\text{sen } x) = x$
- $\text{Sen}(\text{Arcsen } x) = x$
- $\text{Arccos}(\cos x) = x$
- $\cos(\text{Arccos } x) = x$
- $\text{Arctg}(\text{tg}(x)) = x$
- $\text{tg}(\text{Arctg}(x)) = x$
- $\text{Sen}(\text{arccos } x) = \sqrt{1-x^2}$
- $\text{arccos}(\text{sen } x) = \sqrt{1-x^2}$
- $\cos(\text{arcsen } x) \cdot \sqrt{1-x^2}$
- $\text{Arcsen}(\cos x) = \sqrt{1-x^2}$

• Ecuaciones Trigonometricas

1) $\text{Sen } x = a \rightsquigarrow \{\alpha + 2\pi k,\ \pi - \alpha + 2\pi k\}\ k \in \mathbb{Z}$
2) $\cos x = a \rightsquigarrow \{\alpha + 2\pi k,\ -\alpha + 2\pi k\}\ k \in \mathbb{Z}$
3) $\text{tg } x = a \rightsquigarrow \{\alpha + \pi k\}\ k \in \mathbb{Z}$

ej: $5\cos^2 x = 4 - 3\text{sen}^2 x$

$3\text{sen}^2 x + 3\cos^2 x + 2\cos^2 x = 4$

$3(1) + 2\cos^2 x = 4$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

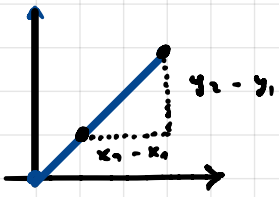
$\cos x = \pm\frac{\sqrt{2}}{2}$ → $\frac{\pi}{4}$: $\{\frac{\pi}{4} + 2\pi k,\ -\frac{\pi}{4} + 2\pi k\}\ k \in \mathbb{Z}$

→ $\frac{3\pi}{4}$: $\{\frac{3\pi}{4} + 2\pi k,\ -\frac{3\pi}{4} + 2\pi k\}$

• Tema 4 : Geometría Analítica

• **Distancia** : $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ $\longrightarrow$ $d(P_2, P_1) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

• **Recta** :



① **Pendiente** : $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

② **Ecuación** : $y - y_1 = m(x - x_1)$, $P(x_1, y_1)$

↳ **Ec. Lineal**: $Ax + By + C = 0$

• **Distancia $P_{(x,y)}$ → Recta** : $d(P, R) = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$, $P(x_1, y_1)$, $R: Ax + By + C = 0$

• **Rectas Paralelas** : $m_1 = m_2$ ↘ $R_1 \parallel R_2$

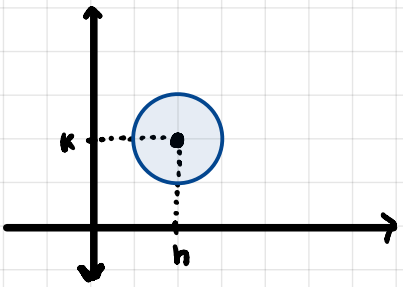
• **Rectas Perpendiculares** : $m_1 \cdot m_2 = -1$ ↘



• **$P_{(x,y)}$ Intersección** : $R_1 = R_2 \rightarrow Ax + By + C = Ax_1 + By_1 + C$

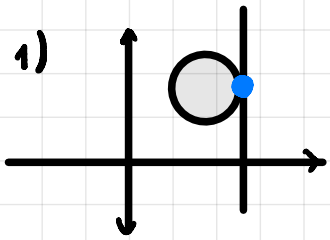
• Secciones Cónicas :

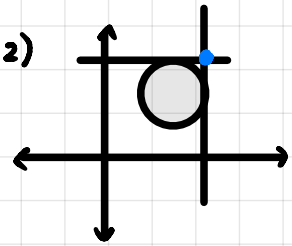
① Circunferencia : $(x-h)^2 + (y-k)^2 = r^2$

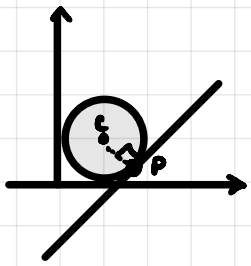


forma General ⇔ $x^2 + y^2 + Dx + Ey + F = 0$

• Recta tangente :







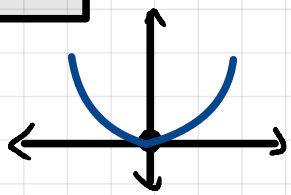
$R(C,P) \Rightarrow m = \frac{y_1 - k}{x_1 - h}$

$(h,k)\ (x_1,y_1)$

$m_1 \cdot m_2 = 1 \quad m_2 = \frac{h-x_1}{k-y_1}$

$$E_c : y = \frac{h-x_1}{k-y_1}(x-x_1) + y_1$$

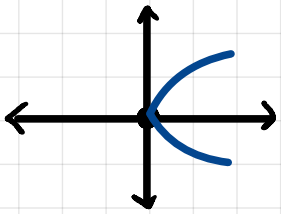
② Parabola : $(x-h)^2 = 4P(y-k)$ →



$d(P,F) = d(P,d)$

Foco directriz

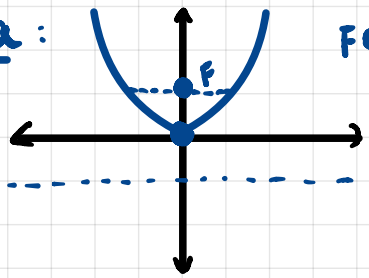
$(y-k)^2 = 4P(x-h)$ →



forma General ⇔ $Ax^2 + Dx + Ey + F = 0$

$By^2 + Dx + Ey + F = 0$

• Grafica :



Foco $(0,P)$. $LR = |4P|$

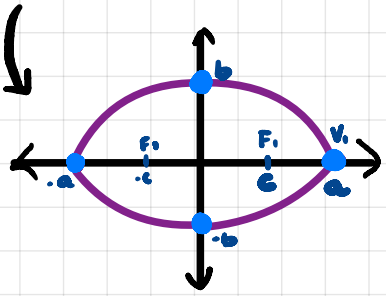
$d = -P$

③ Elipse : $\{ d(P,F_1) + d(P,F_2) = 2a \}$

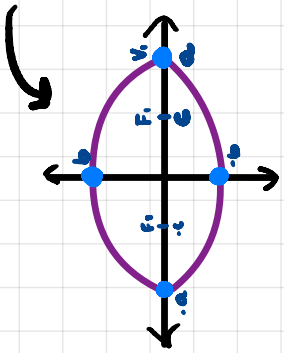
$2a > 2c \rightarrow d(F_1,F_2)$

$a > c$

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



- $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$



$c^2 = a^2 - b^2$

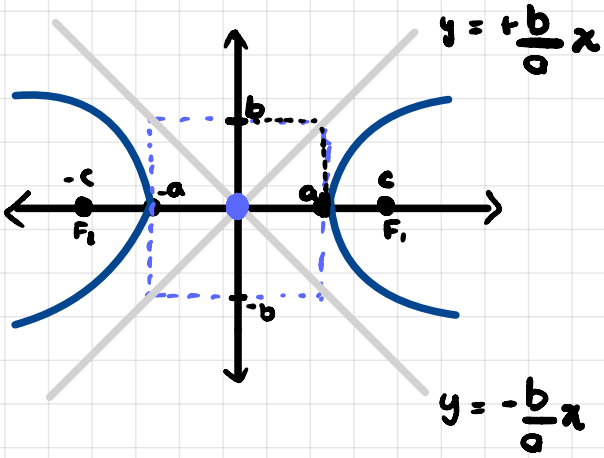
- excent = $e = \frac{c}{a}$

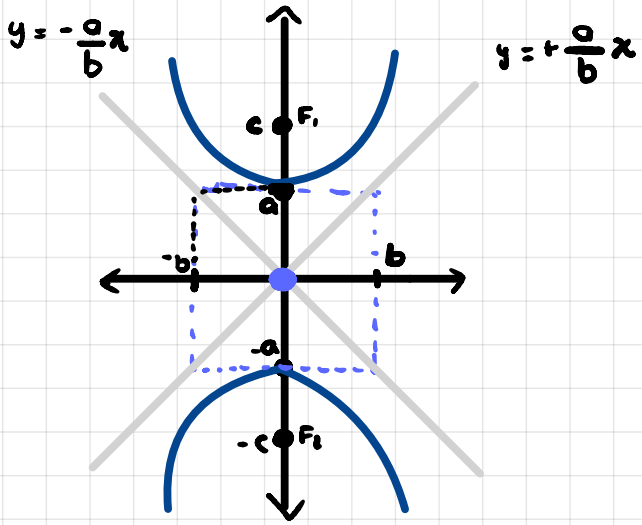
④ Hiperbola :

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

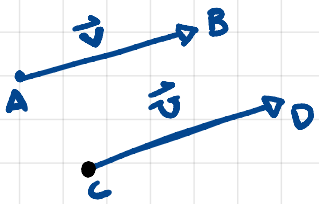
$c^2 = a^2 + b^2$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



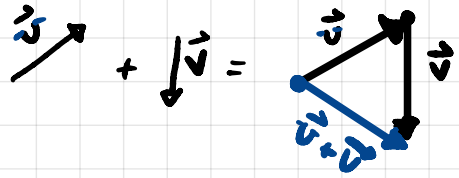


• Vectores :

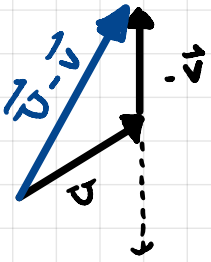


$\vec{v} = \vec{u} \Leftrightarrow$ • misma Dirección
• misma Magnitud

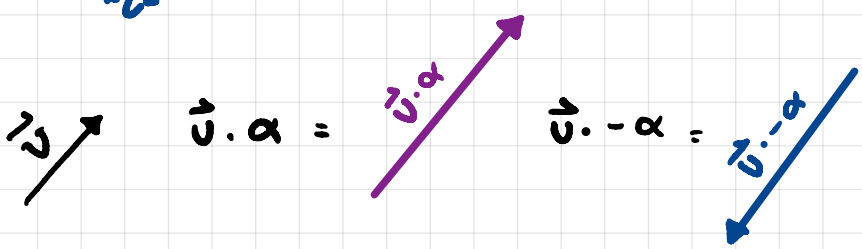
1) **Suma** :



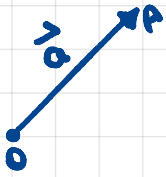
2) **Resta** : $\vec{u} - \vec{v}$:



3) **Multiplo Escalar** :

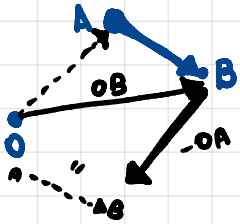


4) **Vector Posición** :

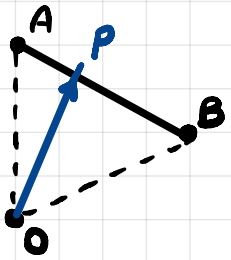


$\vec{OA} = \vec{a}$

5) **Relación Fundamental** : $\vec{AB} = \vec{b} - \vec{a} = \vec{OB} - \vec{OA}$



6) **División Segmento** :



$\Leftrightarrow$

$AP = \lambda PB$

$\vec{OP} - \vec{OA} = \lambda(\vec{OB} - \vec{OP})$

$\vec{P} - \vec{a} = \lambda(\vec{B} - \vec{P})$

$\vec{P} - \vec{A} = \lambda\vec{B} - \lambda\vec{P}$

$\vec{P} + \lambda\vec{P} = \vec{A} + \lambda\vec{B}$

$\vec{P}(1+\lambda) = \vec{A} + \lambda\vec{B}$

$$\vec{P} = \frac{\vec{A} + \lambda\vec{B}}{1+\lambda}$$

7) **Combinación Lineal**

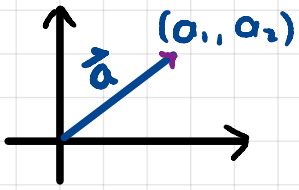
$\vec{v} = x_1\vec{a}_1 + x_2\vec{a}_2 + \dots\dots + x_n\vec{a}_n$

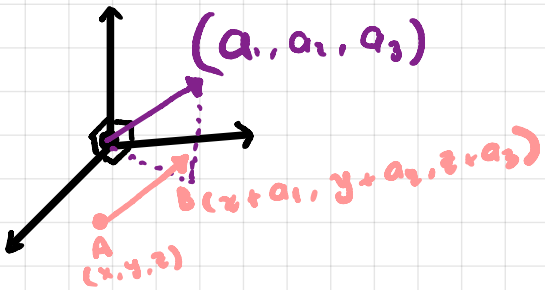
8) **Independencia Lineal** : $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$ son indep Lineal

$\Leftrightarrow x_1\vec{a}_1 + x_2\vec{a}_2 + \dots + x_n\vec{a}_n = 0$

9) **BASE** : si $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$ son Lin. indep y $\forall \vec{a}_n$ se puede escribir como Comb Lineal

- **Componente Vectores:**





- **Vector Posición:** Vector del origen al punto $P(x,y)$

- **Representación:** $\overrightarrow{AB} = \vec{b} - \vec{a} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

- **Magnitud** (Longitud): $\|a\|_{2D} = \sqrt{x^2 + y^2}$, $\|a\|_{3D} = \sqrt{x^2 + y^2 + z^2}$

- **Propiedades:**
  1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
  2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
  3. $\vec{a} + 0 = \vec{a}$
  4. $\vec{a} - \vec{a} = 0$
  5. $c(\vec{a} + \vec{b}) = c \cdot \vec{a} + c \cdot \vec{b}$
  6. $(c + d)\vec{a} = c \cdot \vec{a} + d \cdot \vec{a}$

- **Vectores Unitarios:** $\|\vec{v}\| = 1$

$\hat{i} = (1, 0, 0)$ $\hat{j} = (0, 1, 0)$ $\hat{k} = (0, 0, 1)$

$\vec{a} = (a_1, a_2, a_3) = (a_1, 0, 0) + (0, a_2, 0) + (0, 0, a_3)$

$= a_1 \cdot \hat{i} + a_2 \cdot \hat{k} + a_3 \hat{j}$

- $u = \dfrac{\vec{a}}{\|a\|}$ } Misma dirección que $\vec{a}$, $a \neq 0$

- **Producto punto:** $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

- **Propiedades:**
  1. $\vec{a} \cdot \vec{a} = \|a\|$
  2. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
  3. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a}\vec{c}$

  - $(c \cdot \vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b})$
  - $0 \cdot \vec{a} = 0$

- **Angulo Vectores:** θ entre $\vec{a}$ y $\vec{b}$

$$\vec{a} \cdot \vec{b} = \|a\| \|b\| \cos\theta$$

- **Vectores Perpendiculares:** $\vec{A} \cdot \vec{B} \perp \iff \vec{A} \cdot \vec{B} = 0$ $\theta = \frac{\pi}{2}$

- **Producto Cruz**

  - $\vec{a} = (a_1\ a_2\ a_3)$
  - $\vec{b} = (b_1\ b_2\ b_3)$

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1)$$

↳ **Determinantes:**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

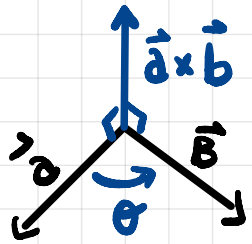
$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

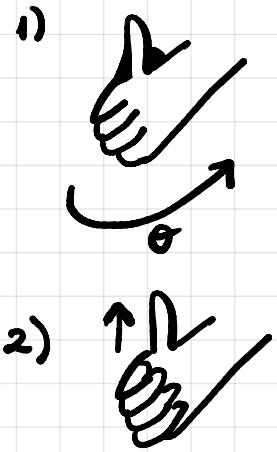
- **Producto Cruz:**

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

- **Teorema:** $\vec{a} \times \vec{b}$ es un Vector Perpendicular

- **Regla Mano Derecha**

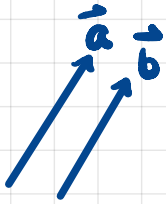




- **Angulo entre Vectores**

$$\|a \times b\| = \|a\| \cdot \|b\| \cdot \operatorname{sen}\theta$$

- **Vectores Paralelos:**



$\Leftrightarrow \vec{a} \times \vec{b} = 0$